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SHORT COMMUNICATIONS

A STUDY ON A NEW MECHANICAL MODEL FOR FOUNDATIONS AND ITS ELASTIC SETTLEMENT RESPONSE

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SUMMARY

In the present paper, a new foundation model has been proposed by introducing a stretched rough elastic membrane in the Pasternak shear layer sandwiched between two spring layers which is an extension of Kerr model. Considering the equilibrium of different elements, the equations governing the elastic settlement response of the model are derived. Finite difference scheme has been employed to solve the governing equations. The parametric studies carried out show the effect of several parameters on the elastic settlement response of the model. The proposed model is well suited for idealizing the behavior of geosynthetic-reinforced granular fill — soft soil system besides other applications.

KEY WORDS: compressibility; elastic settlement; geosynthetic-reinforced soil; mechanical foundation model; prestressing

1. INTRODUCTION

It is a common practice to idealize the behaviour of various materials by mechanical models in many fields of engineering. The earliest and simplest model, proposed by Winkler,[1] is a one-parameter model consisting of closely spaced, independent linear springs. In spite of its several limitations, this model has been extensively used both in the theory and in the engineering practice. Many researchers have tried to improve this model by providing continuity in springs, that is, by attaching different layers to the springs such as the smooth elastic membrane,[2] the elastic beam or plate[3] and the incompressible shear beam or plate.[4] All these models are defined as the two-parameter models as these models have two parameters to represent the characteristics of the material. Kerr[5] and Selvadurai[6] presented excellent surveys of the existing models. Kerr[5] concluded that the Pasternak model, consisting of the Winkler springs provided with shear interactions, is mechanically the most logical extension of the Winkler model and analytically the next higher approximation. As a generalization of the Pasternak model, Kerr suggested a three-parameter model consisting of two elastic spring layers interconnected by an elastic shear layer. This model has several advantages over the Pasternak model as pointed out by Kerr[7] and Rhines.[8] To simulate the punching shear failure in soils, Rhines[8] extended the Kerr model by including a plastic yielding phenomenon in the shear layer.

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In the recent years, the development of composite materials has provided a new impetus for the development of improved mechanical models. Madhav and Poorooshasb[9] presented a model for the geosynthetic-reinforced soil by developing a new foundation model element using a rough membrane for representing the geosynthetic. To account for the confinement effect of the reinforcement, Madhav and Poorooshasb[10] presented a modified Pasternak model using a variable shear modulus. All these models provide good results under small stresses for which there is no large deformations.[11] The model element proposed by Madhav and Poorooshasb[9] which has a rough elastic membrane embedded in the shear layer has been modified by Shukla and Chandra[12] to include the prestressing effect for representing the prestressed geosynthetic.

The modulus of subgrade reaction approach, in spite of all its limitations, is preferred to analyse many geotechnical engineering problems due to its greater ease of use and substantial saving in computer computation time. The authors have experienced that until the state of the art improves and accurate values of elastic parameters for the soil can be determined, the finite element analysis cannot be a better alternative to this approach.

In the present paper, the Kerr model has been extended by including stretched rough elastic membrane in the shear layer. This new model can have application in several areas of engineering, especially in those situations where the membrane-like material (for example, ropes, wires, metallic sheets, thin rubber pads, geosynthetics, etc.) with or without prestressing is sandwiched between two compressible layers. The load-settlement response of the suggested model has been presented for plane strain conditions of the loading. The elastic settlements in this study are computed by an iterative finite difference scheme. The results have been presented in a non-dimensional form for practical applications.

2. PROPOSED FOUNDATION MODEL AND ITS RESPONSE FUNCTION

Figure 1 shows the proposed mechanical foundation model which consists of the shear layer containing a stretched rough elastic membrane, sandwiched between two spring layers. The membrane divides the shear layer into two parts, one above and the other below it. In practical applications the characteristics of these two layers can be different from each other. The general assumptions are that the membrane is linearly elastic, rough enough to prevent slippage at the interface with shear layer and has no shear resistance. A rigid-perfectly plastic friction model is

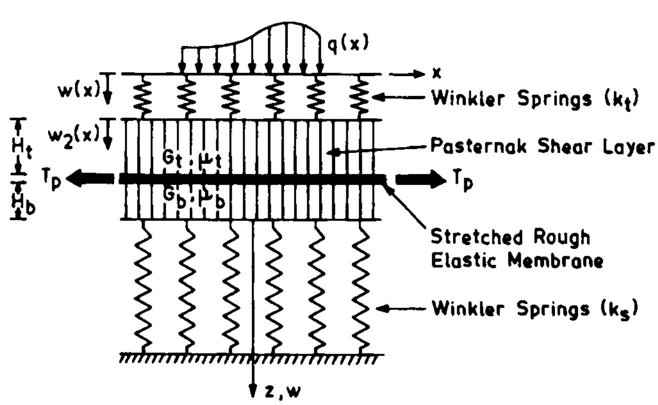

Figure 1. Definition sketch — proposed foundation model

adopted to represent the behaviour of the shear layer–membrane interface. The spring constant is assumed to have a constant value. The analysis is made for a strip loading resting directly on the top of the upper spring layer in plane strain conditions. The equation governing the response of the model has been derived by considering the equilibrium of forces on different elements of the shear layer and the stretched rough elastic membrane as shown in Figure 2. At the shear layer–membrane interface, only the horizontal shear stress transfer mechanism has been considered (Figure 2(b)).

The governing differential equations can be written in the non-dimensional form as

$$(1 + \bar{X}_1/\alpha)q^* - \frac{1}{\alpha}\{G_t^* + \bar{X}_2(T_p^* + T^*)\cos\theta + \bar{X}_2 G_b^*\}\frac{\mathrm{d}^2 q^*}{\mathrm{d}X^2}$$

$$= \bar{X}_1 W - \{G_t^* + \bar{X}_2(T_p^* + T^*)\cos\theta + \bar{X}_1 G_b^*\}\frac{\mathrm{d}^2 W}{\mathrm{d}X^2} \tag{1}$$

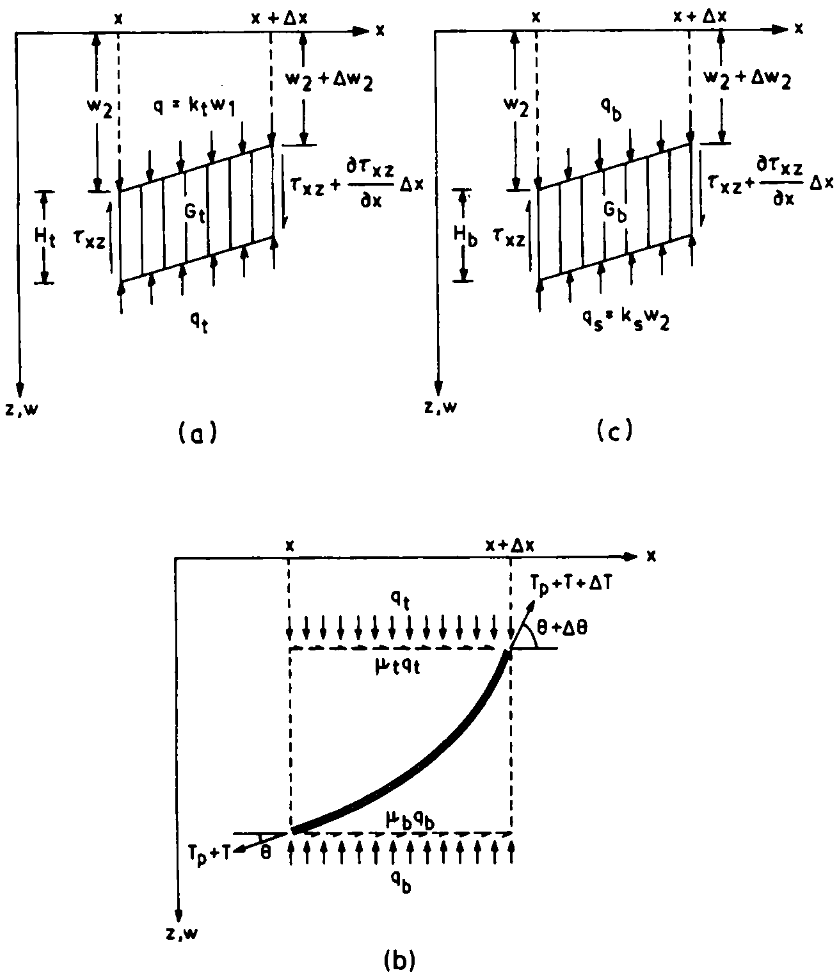

Figure 2. Definition sketch: (a) forces on the upper shear layer element; (b) forces on the stretched rough elastic membrane element; (c) forces on the lower shear layer element

and

$$\frac{dT^*}{dX} = -\bar{X}_3\left\{q^* + G_t^*\left(\frac{d^2W}{dX^2} - \frac{1}{\alpha}\frac{d^2q^*}{dX^2}\right)\right\} - \bar{X}_4\left\{(W - q/\alpha) - G_b^*\left(\frac{d^2W}{dX^2} - \frac{1}{\alpha}\frac{d^2q^*}{dX^2}\right)\right\} \quad (2)$$

where $X = x/B$ $W = w/B$, $G_t^* = G_tH_t/k_sB^2$, $G_b^* = G_bH_b/k_sB^2$, $q^* = q/k_sB$, $T_p^* = T_p/k_sB^2$, $T^* = T/k_sB^2$, $\alpha(= k_t/k_s)$ is the spring constant ratio, q is the applied load intensity, k_s and k_t are spring constants per unit area for lower and upper spring layers, respectively, G_b ad G_t are shear moduli for the lower and upper shear layers, respectively, H_b and H_t are thicknesses of lower and upper shear layers, respectively, μ_b and μ_t are interface friction coefficients at the bottom and top faces of the membrane, respectively, w is the total vertical displacement, T is the tensile force per unit length mobilized in the membrane, T_p is the pretension per unit length applied to the membrane, B is the half-width of the strip loading, and θ is the slope of the membrane.

Non-dimensional parameters $\bar{X}_1$, $\bar{X}_2$, $\bar{X}_3$, $\bar{X}_4$ are given by

$$\bar{X}_1 = \frac{1 - \mu_b \tan\theta}{1 + \mu_t \tan\theta} \quad (3)$$

$$\bar{X}_2 = \frac{1}{1 + \mu_t \tan\theta} \quad (4)$$

$$\bar{X}_3 = \mu_t \cos\theta - \sin\theta \quad (5)$$

and

$$\bar{X}_4 = \mu_b \cos\theta + \sin\theta \quad (6)$$

In an attempt to reduce the constants in the proposed model to an absolute minimum, one can assume the value of α(= 3 as suggested by Kerr[7]). With such an assumption the proposed model becomes a four-parameter model defined by k_s, G_t/G_b, μ_t/μ_b and T_p.

3. DETERMINATION OF MODEL PARAMETERS

The determination of model parameters, specially modulus of subgrade reaction (k) and shear parameter (G), can be done along the lines suggested by Selvadurai[6] for mechanical foundation models. The modulus of subgrade reaction is commonly determined from plate load test. It is not a unique property of the soil medium and it gets affected by several factors such as size of footing, shape of footing, embedded depth of footing, and material characteristics of footing. Hence the modulus of subgrade reaction obtained from a plate load test in the field has to be appropriately corrected before using it for solving practical problems.

The model parameters k and G have been related to elastic constants (Young's modulus, E and Poisson's ratio, v) of the soil and several relationships have been suggested in past. For a single layer of thickness H with a linear variation of normal stresses, the two-parameter model of Vlazov and Leontiev[13] provides the following relations:

$$k = \frac{E}{H(1 + v)(1 - 2v)} \quad (7)$$

and

$$G = \frac{EH}{6(1 + v)} \quad (8)$$

The determination of interfacial friction coefficient (μ) between soil and geosynthetic reinforcement is generally made using a modified direct shear test.[14,15] A linear relationship between the interfacial shear strength and the normal stress usually assumed provides the following expression for the interfacial friction coefficient, μ:[16]

$$\mu = C_a/\sigma_v + \tan\delta \tag{9}$$

where C_a is the interface adhesion, δ is the interface friction angle, and σ_v is the stress normal to the surface. This relationship can be used directly to evaluate the friction coefficients at the soil–geosynthetic interface. However, in many cases the relationship between the interface shear strength and the normal stress is not linear. In such cases, a non-linear relationship proposed by Giroud *et al.*[17] may be used to evaluate interfacial friction coefficients.

It should be noted that the use of empirical relationships as well as experimental techniques described above are only some of the possible means to estimate the model parameters.

4. NUMERICAL SOLUTION

To observe the settlement response of the proposed model, equations (1) and (2) are solved iteratively by finite difference scheme for specified loading and boundary conditions. Writing equations (1) and (2) in finite difference form, for an interior mode i, within the domain as specified, one gets

$$\begin{aligned}(1 + \bar{X}_{1i}/\alpha)q_i^* &- \frac{1}{\alpha}\{G_t^* + \bar{X}_{2i}(T_p^* + T_i^*)\cos\theta_i + \bar{X}_{2i}G_b^*\}\left.\frac{d^2q^*}{dX^2}\right|_i \\ &= \bar{X}_{1i}W_i - \{G_t^* + \bar{X}_{2i}(T_p^* + T_i^*)\cos\theta_i + \bar{X}_{1i}G_b^*\}\left.\frac{d^2W}{dX^2}\right|_i\end{aligned} \tag{10}$$

and

$$\begin{aligned}T_i^* = T_{i+1}^* + (\Delta X/2)\Bigg[\bar{X}_{3i}\Bigg\{&(q_i^* + q_{i+1}^*) + G_t^*\left(\left(\left.\frac{d^2W}{dX^2}\right|_i + \left.\frac{d^2W}{dX^2}\right|_{i+1}\right)\right. \\ &\left.\left. - \frac{1}{\alpha}\left(\left.\frac{d^2q^*}{dX^2}\right|_i + \left.\frac{d^2q^*}{dX^2}\right|_{i+1}\right)\right)\right\} + \bar{X}_{4i}\Bigg\{(W_i + W_{i+1}) - (q_i^* + q_{i+1}^*)/\alpha \\ &- G_b^*\left(\left(\left.\frac{d^2W}{dX^2}\right|_i + \left.\frac{d^2W}{dX^2}\right|_{i+1}\right) - \frac{1}{\alpha}\left(\left.\frac{d^2q^*}{dX^2}\right|_i + \left.\frac{d^2q^*}{dX^2}\right|_{i+1}\right)\right)\Bigg\}\Bigg]\end{aligned} \tag{11}$$

In order to minimize the numerical error, average values of q^*, W, d^2q^*/dX^2, and d^2W/dX^2 for each element are taken in equation (11).

The solutions are obtained for uniform flexible strip loading. Hence, the loading conditions considered are given by

$$q_i^*(X) = \begin{cases} q_0^* & \text{for } |X| \leqslant 1{\cdot}0 & (12a) \\ 0{\cdot}0 & \text{for } |X| > 1{\cdot}0 & (12b) \end{cases}$$

where $q_0^*(= q_0/k_s B)$ is the non-dimensionalized uniform load intensity.

Boundary conditions considered at the centre and at the edge of the membrane/shear layer are as follows:

$$\mathrm{d}W/\mathrm{d}X = 0{\cdot}0 \quad \text{at } X = 0{\cdot}0 \tag{13a}$$

$$\mathrm{d}W/\mathrm{d}X = 0{\cdot}0 \quad \text{at } X = L/B \tag{13b}$$

$$T^* = 0{\cdot}0 \quad \text{at } X = L/B \tag{13c}$$

where L is the half-width of the membrane/shear layer.

The first boundary condition directly comes from the symmetry of the problem about the centre of the loaded region. The second boundary condition comes from the fact that the membrane is generally horizontal at its edge where it is free, or fixed. The third boundary condition implies that the frictional resistance mobilized over the length of the membrane is sufficient to balance tensile force in it and hence tensile force at the free edge of the membrane is taken as zero. In field, the non-zero values of mobilized tensile force may occur at the edge of the membrane if frictional resistance is not sufficient to balance the tensile force in it. The influence of non-zero values of T^* can be studied by specifying the particular values of T^* at $X = L/B$ as the boundary condition in the suggested model.

5. RESULTS AND DISCUSSION

Based on the above formulation, results were obtained using the HP-9000/850 computer system. Due to the symmetry of the problem analysed, only half region of the problem ($X \geqslant 0$) is considered. The solutions are obtained with a convergence criterion as

$$\left|\frac{W_i^j - W_i^{j-1}}{W_i^j}\right| \times 100\% < \varepsilon_s \tag{14}$$

for all i, where j and $j-1$ are the present and previous iterations, and ε_s is the specified tolerance which is considered to be 0.0001 in the present study. The ranges of parameters studied are: load intensity q^*, 0·0–1·0; shear parameter G_t^* or G_b^*, 0·01–1·0; interface friction coefficient μ_t or μ_b, 0·1–1·0; prestress in the rough elastic membrane T_p^*, 0·0–1·0; spring constant ratio, α, 3–50.

Figure 3 illustrates the comparison of the typical settlement profile obtained by the present model ($\alpha = 10$, $T_p^* = 0$) with those obtained from the several existing models. It is observed that there is a sharp discontinuity in the profiles of Winkler, Kerr and the proposed models at the edge of the loaded region. This is due to the inherent limitation of the Winkler springs. In addition to the several advantages as pointed out by Kerr[7] and Rhines,[8] the upper Winkler springs in the present model can also idealize the compressibility of the granular fill in geosynthetic-reinforced granular fill–soft soil system extensively used in the present day practice of geotechnical engineering as a foundation for many civil engineering structures. It can also be noted here that the Pasternak and Kerr models predict the same settlement at any location outside the loaded region.

Figure 4 shows the effect of spring constant ratio on the settlement response of the proposed model. It is noticed that the settlement at any location decreases as the spring constant ratio increases. For example, for the increase of spring constant ratio from 3 to 50, the settlement reduction at the centre of the loaded region for the set of parameters studied is observed to be 27·85 per cent, whereas at the edge of the loaded region, the reduction in settlement is 43·53 per cent. It can also be noted that the sudden drop in the settlement at the edge of the loaded region is more for lower spring constant ratio. For example for spring constant ratio, $\alpha = 3$, the sudden drop in the settlement at the edge of the loaded region is observed to be 35·25 per cent, whereas

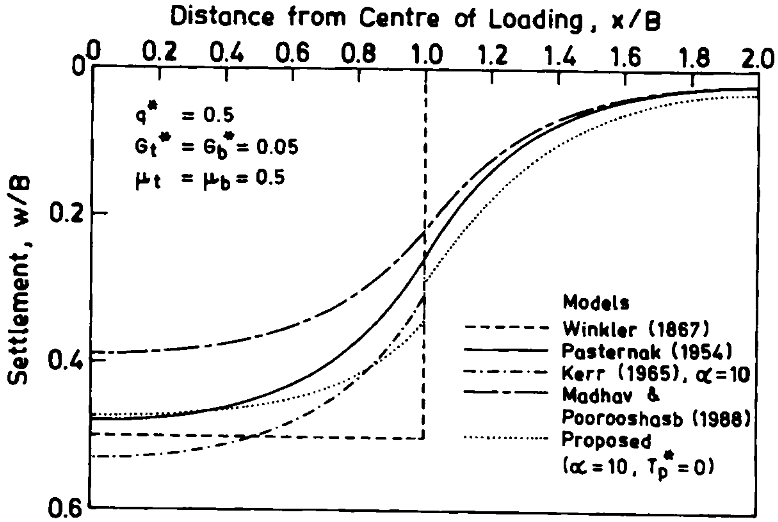

Figure 3. Settlement–distance profiles — comparison of various existing foundation models

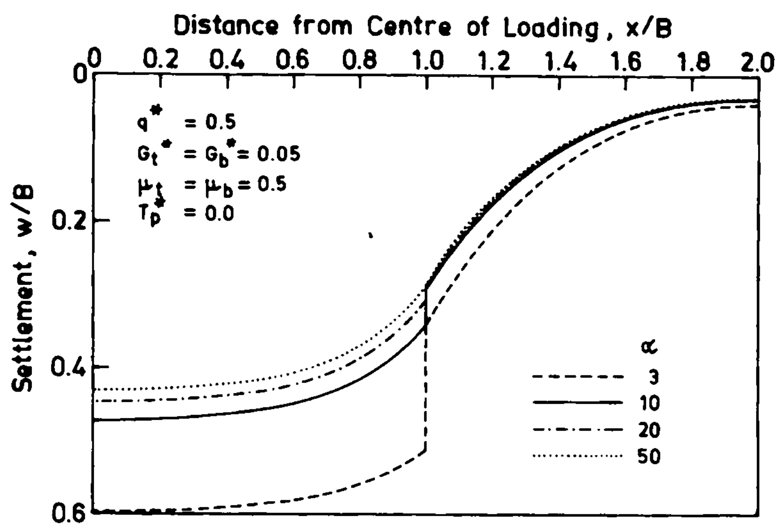

Figure 4. Settlement–distance profiles — effect of spring constant ratio

for $\alpha = 50$, the drop is 8·00 percent. These results indicate that the proper idealization of the settlement response of the model depends much on the assumed value of the spring constant ratio.

The effect of load intensity on the settlement–distance profiles is shown in Figure 5. It is noticed that the discontinuity in the settlement profile at the edge of the loaded region is relatively large for higher non-dimensional load intensity. This indicates that the value of modular ratio must be selected appropriately in such situation.

Figure 6 shows the effect of prestressing the rough elastic membrane on the settlement response of the proposed model. It is observed that with increase in prestress in the rough elastic

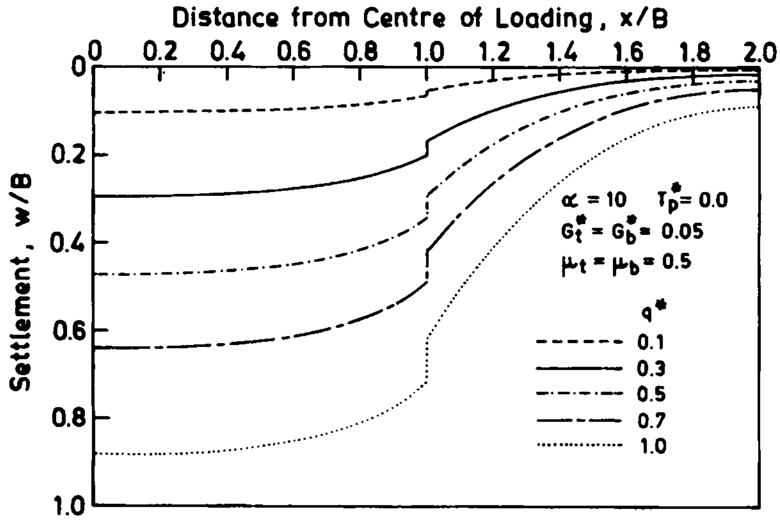

Figure 5. Settlement-distance profiles—effect of load intensity

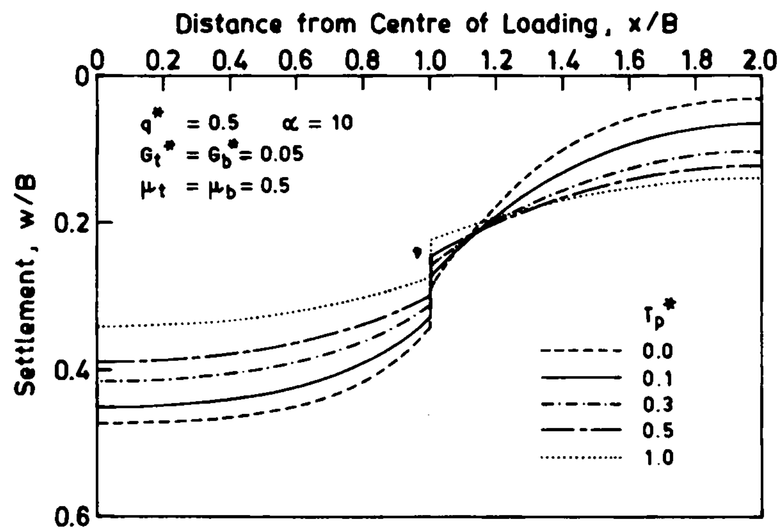

Figure 6. Settlement-distance profiles—effect of prestressing

membrane, the settlement reduces significantly within the loaded region, whereas slightly beyond the loaded region, the increase in the settlement occurs. The settlement reductions for non-dimensional prestress values of 0·1, 0·3, 0·5 and 1·0 compared with no prestressing ($T_p^* = 0{\cdot}0$) are 4·87, 12·29, 17·80 and 27·97 per cent at the centre of the loaded region, whereas at the edge, the reductions are 3·81, 8·80, 12·32 and 19·65 per cent, respectively. This indicates that the reduction in the settlement due to prestressing is more significant at the centre of the loaded region than that at the edge of the loaded region. Similar trend has also been observed by Shukla and Chandra.[12]

The effect of shear parameter on the settlement response of the suggested model is shown in Figure 7. It is noted that the settlement reduces with the increase of shear parameter within the

loaded region, whereas slightly beyond the loaded region, the increase in the settlement occurs. The settlement reductions for the shear parameters ($G_t^* = G_b^*$) of 0·1, 0·5 and 1·0 compared with the case of $G_t^* = G_b^* = 0{\cdot}01$ are 9·84, 30·72 and 43·17 per cent at the centre of the loaded region, whereas at the edge of the loaded region, the reductions are 16·50, 29·38 and 38·66 per cent, respectively. It can be noticed here that for the higher shear parameters (that is, for the higher thickness of the shear layer or the higher shear modulus) the order of decrease in the settlement is more at the centre than that at the edge of the loaded region.

Figure 8 shows the effect of interface friction coefficient on the settlement response of the proposed model. It is observed that as the interface friction coefficient increases, the settlement

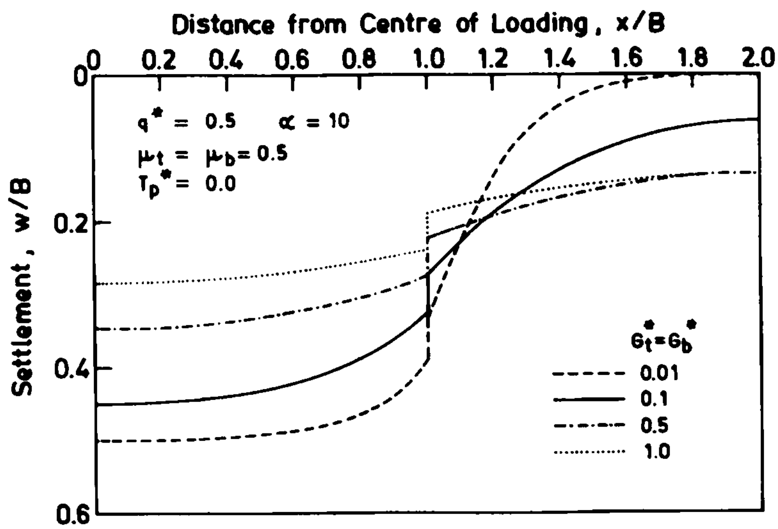

Figure 7. Settlement–distance profiles—effect of shear parameters

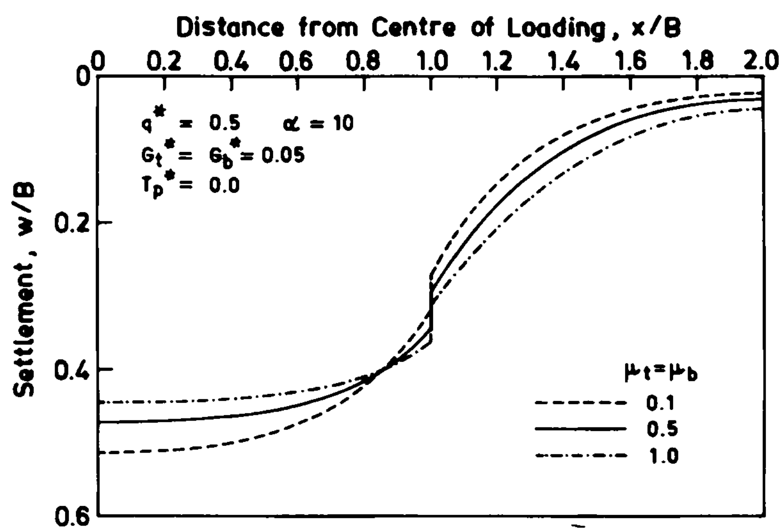

Figure 8. Settlement–distance profiles—effect of interface friction coefficients

reduces within most of the loaded region except near the edge of the loaded region, whereas the settlement increases beyond the loaded region. For an increase of the interface friction coefficient ($\mu_t = \mu_b$) from 0·1 to 0·5, the settlement reduces by 8·17 per cent at the centre of the loaded region, whereas at the edge of the loaded region, the settlement increases by 7·57 per cent.

6. CONCLUSIONS

The proposed model is fairly general and can be applied for idealizing the behaviour of several engineering materials. This model is well suited for predicting the behaviour of geosynthetic-reinforced granular fill—soft soil system as it considers the compressibility of the granular fill which has been generally disregarded in most of the studies carried out so far.

The response function of the model is given by equations (1) and (2). The settlement response of the model is governed by several parameters such as the spring constant ratio, the prestress is the rough elastic membrane, the shear parameter and the interface friction coefficients. The parametric studies reveal that even a small variation in these parameters results in significant change in the settlement response. For example, as the spring constant ratio is increased by approximately 3 times, the decrease in the settlement is about 20 per cent at the centre of the loaded region, whereas at the edge, the decrease is about 33 per cent. This fact suggests the need of the proper estimation of these parameters while using the suggested model. The settlement response of the proposed model for several prestress values has a trend similar to the one reported in the literature.

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